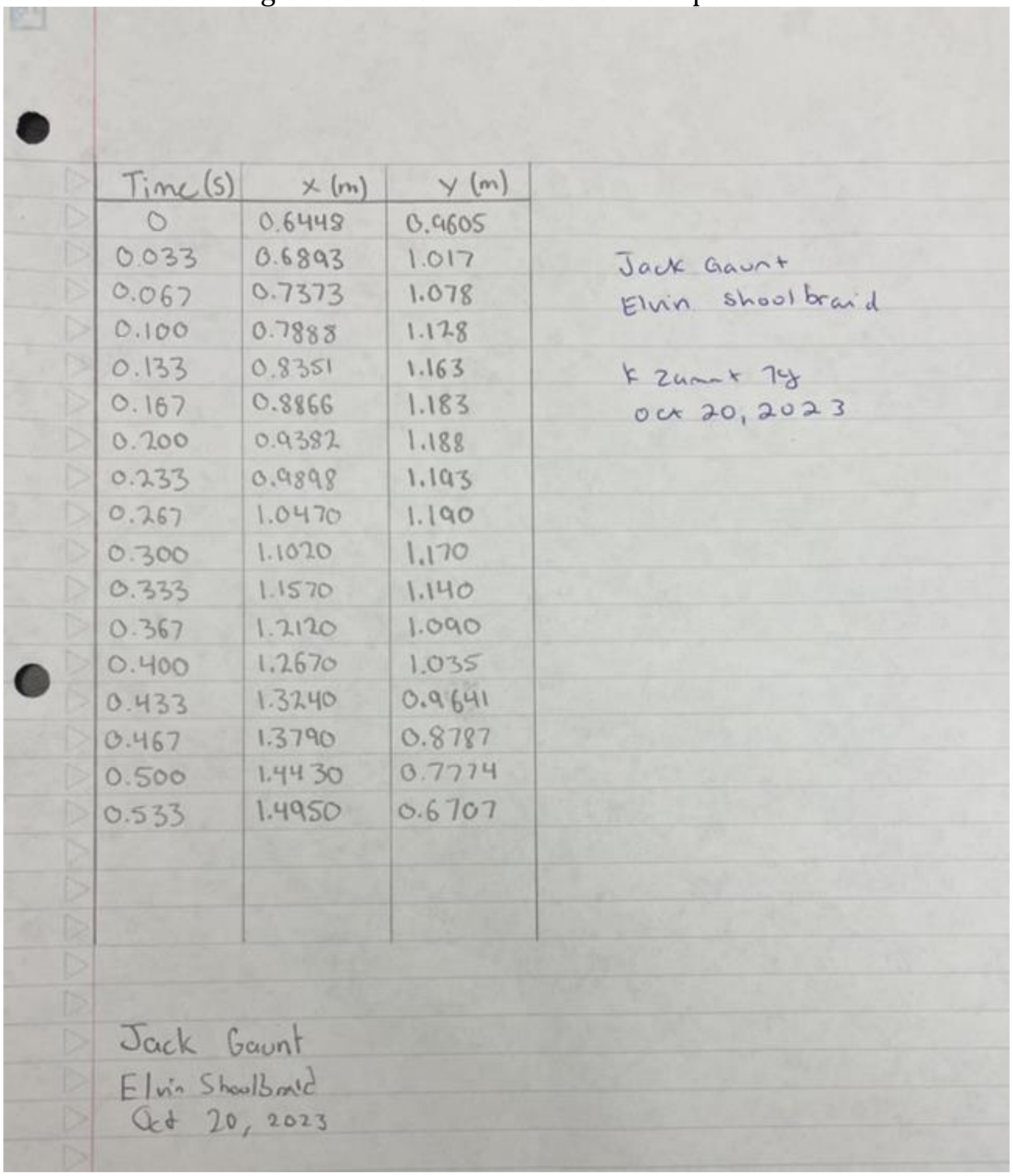
Project 4 Worksheet

1. Include a photo of your raw work, including your TA's signature. Failure to include this data will result in a grade of zero for the entire lab report.



Time (s)	x (m)	y (m)
0	0.6448	0.9605
0.033	0.6893	1.017
0.067	0.7373	1.078
0.100	0.7888	1.128
0.133	0.8351	1.163
0.167	0.8866	1.183
0.200	0.9382	1.188
0.233	0.9898	1.193
0.267	1.0470	1.190
0.300	1.1020	1.170
0.333	1.1570	1.140
0.367	1.2120	1.090
0.400	1.2670	1.035
0.433	1.3240	0.9641
0.467	1.3790	0.8787
0.500	1.4430	0.7774
0.533	1.4950	0.6707

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2. Create a table to show all your data as described in the Analysis & Submission section of the lab manual. Explain which method was used for calculating the velocity components and show a sample calculation.

Time (s)	X (m)	Z (m)	Vx (m/s)	Vz (m/s)
0.000	0.645	0.961	/	/
0.033	0.689	1.02	1.38	1.75
0.067	0.737	1.08	1.49	1.66
0.100	0.789	1.13	1.48	1.29
0.133	0.835	1.16	1.46	0.821
0.167	0.887	1.18	1.54	0.373
0.200	0.938	1.19	1.56	0.152
0.233	0.990	1.19	1.62	0.030
0.267	1.05	1.19	1.67	-0.343
0.300	1.10	1.17	1.67	-0.758
0.333	1.16	1.14	1.64	-1.19
0.367	1.21	1.09	1.64	-1.57
0.400	1.27	1.04	1.70	-1.91
0.433	1.32	0.964	1.67	-2.33
0.467	1.38	0.879	1.78	-2.79
0.500	1.44	0.777	1.76	-3.15
0.533	1.50	0.671	/	/

Note: unrounded values used for All calculations in this report

For all i's between but not including 1 and 17 (for which i – 1 or i + 1 don't exist, respectively), the central difference method was used to calculate the velocity components:

$$V_i = \frac{X(t_{i+1}) - X(t_{i-1})}{t_{i+1} - t_{i-1}}$$

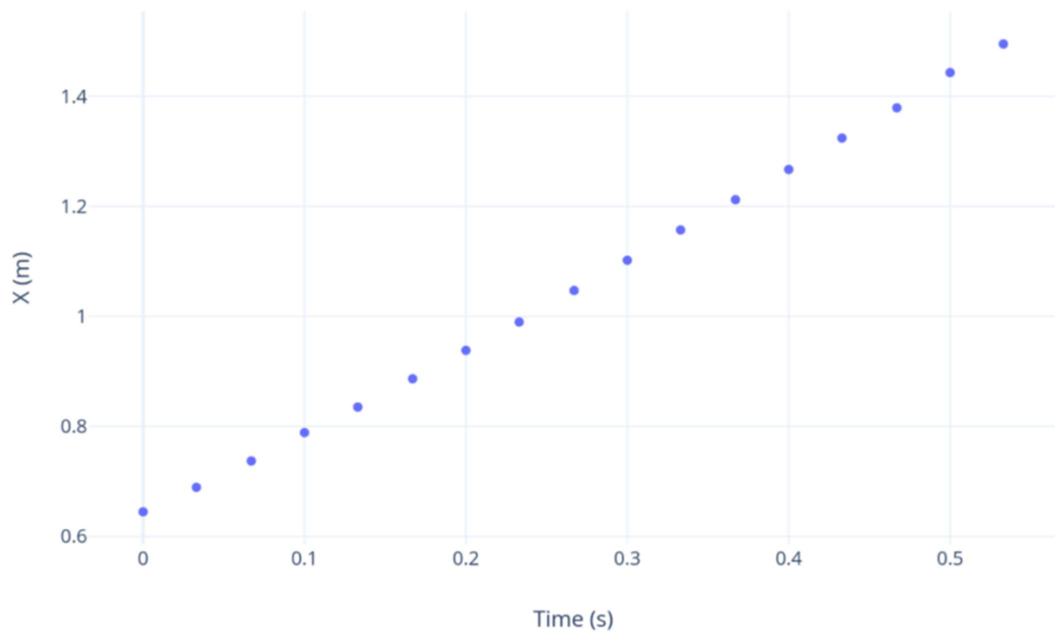
for example, $i = 2$:

$$V_2 = \frac{X(t_3) - X(t_1)}{t_3 - t_1} = \frac{0.7373\text{m} - 0.6448\text{m}}{0.067\text{s} - 0\text{s}} = 1.38\,\text{m/s}$$

rounded to 3 s.f for display

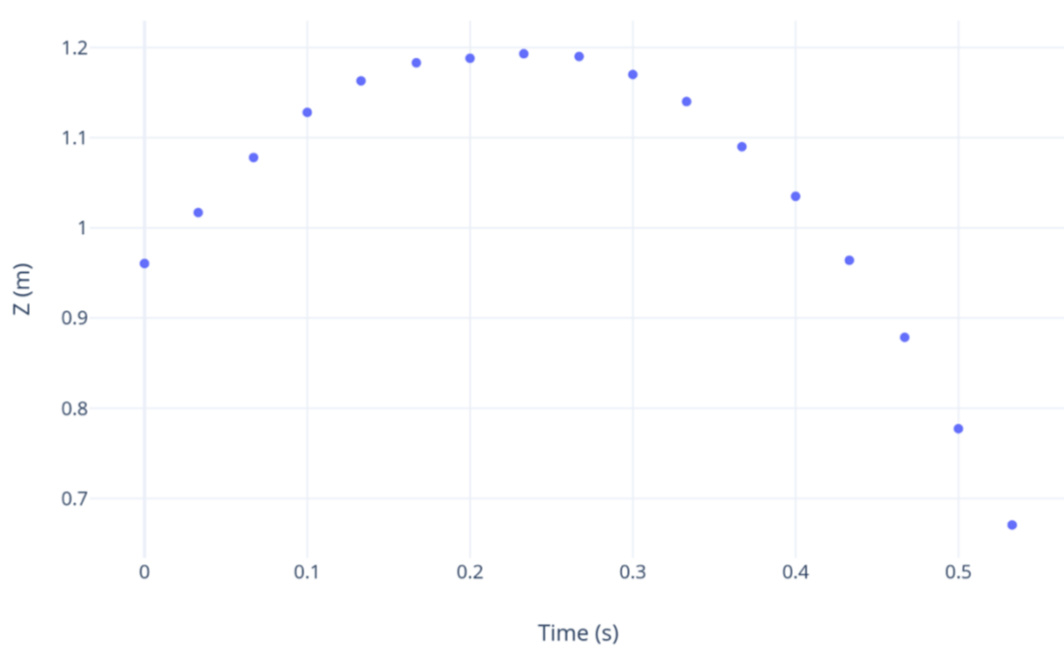
3. Create the four graphs as described in the Analysis & Submission section of the manual.

X (m) vs Time (s):



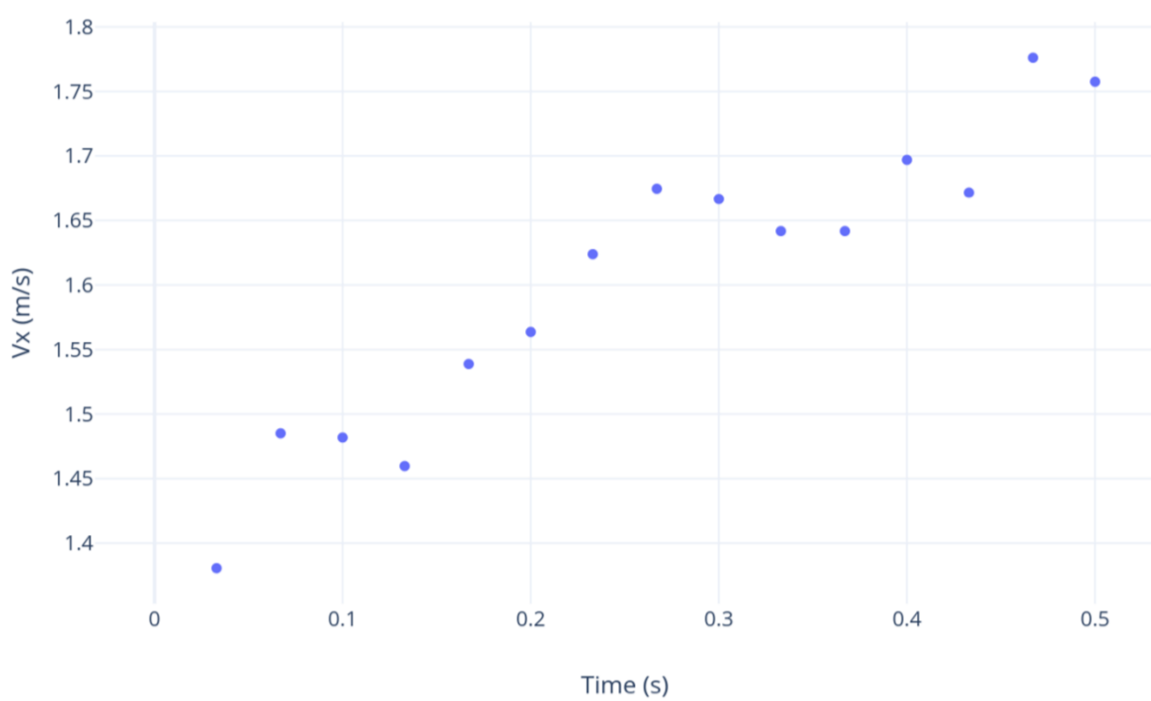
Data ranges: Time: t1:t17 (0s ≤ t ≤ 0.533s), X: x1:x17 (0.6448m ≤ x ≤ 1.495m)

Z (m) vs Time (s):



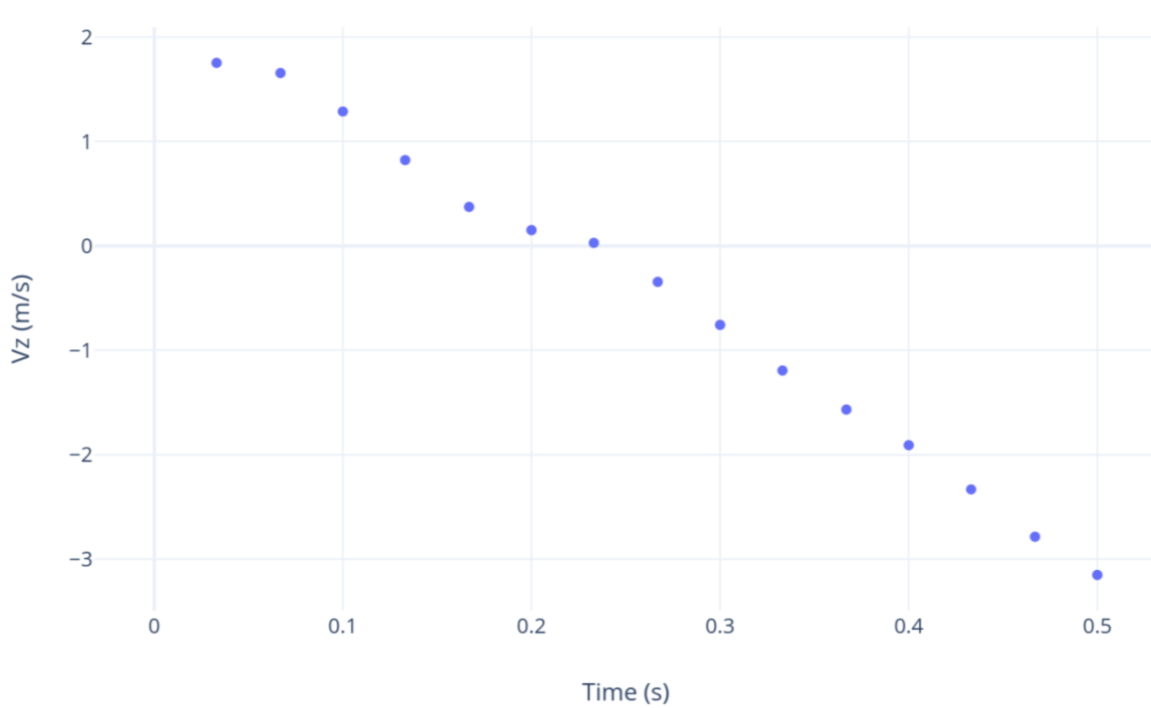
Data ranges: Time: t1:t17 (0s ≤ t ≤ 0.533s), Z: z1:z17 (0.6707m ≤ z ≤ 1.193m)

Vx (m/s) vs Time (s):



Data ranges: Time: t2:t16 ($0.033s \le t \le 0.5s$), Vx: Vx2:Vx16 ($1.381m/s \le Vx \le 1.776m/s$)

Vz (m/s) vs Time (s):



Data ranges: Time: t2:t16 ($0.033s \le t \le 0.5s$), Vz: Vz2:Vz16 ($-3.152m/s \le Vz \le 1.754m/s$)

4. Create a table for the least squares analysis of the vx vs t data, as described in the Analysis & Submission section of the lab manual.

t	**Vx**	**t^2**	**Vx^2**	**Vxt**
0.033	1.38	0.001	1.91	0.046
0.067	1.49	0.004	2.21	0.099
0.100	1.48	0.010	2.20	0.148
0.133	1.46	0.018	2.13	0.194
0.167	1.54	0.028	2.37	0.257
0.200	1.56	0.040	2.44	0.313
0.233	1.62	0.054	2.64	0.378
0.267	1.67	0.071	2.80	0.447
0.300	1.67	0.090	2.78	0.500
0.333	1.64	0.111	2.70	0.547
0.367	1.64	0.135	2.70	0.603
0.400	1.70	0.160	2.88	0.679
0.433	1.67	0.187	2.79	0.724
0.467	1.78	0.218	3.15	0.829
0.500	1.76	0.250	3.09	0.879

Averages:

0.267	1.60	0.0919	2.59	0.443

Note: Only t values for which a corresponding velocity value is defined were included in the least squares analyses, as the equations involved are only defined when both values exist. This also means N = 15 in all the equations.

5. Determine the slope and intercept for the vx vs t data, along with their uncertainties. This should be done by hand using the average column values in the table above.

$$\text{slope} = A = \frac{(\overline{xy}) - (\bar{x})(\bar{y})}{(\overline{x^2}) - (\bar{x})^2} = \frac{(\overline{V_x t}) - (\bar{t})(\bar{V_x})}{(\overline{t^2}) - (\bar{t})^2}$$

$$\text{slope} = \frac{0.442845 - 0.266667 \times 1.604046}{0.091859 - (0.26667)^2} = 0.727746 \ (m/s^2)$$

$$\text{intercept} = B = \frac{(\overline{x^2})(\bar{y}) - (\bar{x})(\overline{xy})}{(\overline{x^2}) - (\bar{x})^2} = \frac{(\overline{t^2})(\bar{V_x}) - (\bar{t})(\overline{V_x t})}{(\overline{t^2}) - (\bar{t})^2}$$

$$\text{intercept} = \frac{0.091859 \times 1.604046 - 0.266667 \times 0.442845}{0.091859 - (0.266667)^2} = 1.409981 \ (m/s)$$

$$\delta_{\text{slope}} = \delta\Delta\sqrt{\frac{1}{N(\overline{x^2} - \bar{x}^2)}} \qquad \delta_{\text{intercept}} = \delta\Delta\sqrt{\frac{\overline{x^2}}{N(\overline{x^2} - \bar{x}^2)}}$$

$$\delta\Delta = \sqrt{\frac{\Delta\text{total}}{N-2}} \longrightarrow \Delta\text{total} = N\left(\overline{y^2} + A^2\,\overline{x^2} + B^2 - 2A\,\overline{xy} - 2B\bar{y} + 2AB\bar{x}\right)$$

$$\Delta\text{total} = N\left(\overline{V_x^2} + A^2\,\overline{t^2} + B^2 - 2A\,\overline{V_x t} - 2B\bar{V_x} + 2AB\bar{t}\right)$$

$$\Delta\text{total} = 15\big(2.58525 + (0.727746)^2(0.091859) + (1.409981)^2 - 2(0.727746)(0.442845) - 2(1.409981)(1.604046) + 2(0.727746)(1.409981)(0.26667)\big) = 0.01945$$

$$\delta\Delta = \sqrt{\frac{0.01945}{13}} = 0.03868$$

$$\delta_{\text{slope}} = (0.03868)\sqrt{\frac{1}{N(\overline{t^2} - \bar{t}^2)}} \qquad \delta_{\text{intercept}} = (0.03868)\sqrt{\frac{\overline{t^2}}{N(\overline{t^2} - \bar{t}^2)}}$$

$$\delta_{\text{slope}} = (0.03868)\sqrt{\frac{1}{15(0.091859 - (0.266667)^2)}} = 0.069335 \ (m/s^2)$$

round to first non-0 digit $= 0.07$

$$\delta_{\text{intercept}} = (0.03868)\sqrt{\frac{0.091859}{15(0.091859 - (0.266667)^2)}} = 0.021014 \ (m/s) = 0.02$$

6. Create a table for the least squares analysis of the vz vs t data, as described in the Analysis & Submission section of the lab manual.

t	**Vz**	**t^2**	**Vz^2**	**Vzt**
0.033	1.75	0.001	3.08	0.058
0.067	1.66	0.004	2.74	0.111
0.100	1.29	0.010	1.66	0.129
0.133	0.821	0.018	0.674	0.109
0.167	0.373	0.028	0.139	0.062
0.200	0.152	0.040	0.023	0.030
0.233	0.030	0.054	0.001	0.007
0.267	-0.343	0.071	0.118	-0.092
0.300	-0.758	0.090	0.574	-0.227
0.333	-1.19	0.111	1.43	-0.398
0.367	-1.57	0.135	2.46	-0.575
0.400	-1.91	0.160	3.64	-0.763
0.433	-2.33	0.187	5.44	-1.01
0.467	-2.79	0.218	7.76	-1.30
0.500	-3.15	0.250	9.93	-1.58

Averages:

0.267	-0.531	0.0919	2.64	-0.362

7. Determine the slope and intercept for the vz vs t data, along with their uncertainties. This should be done by hand using the average column values in the table above.

$$\text{slope} = A = \frac{(\overline{xy}) - (\bar{x})(\bar{y})}{(\overline{x^2}) - (\bar{x})^2} = \frac{(\overline{V_z t}) - (\bar{t})(\bar{V_z})}{(\overline{t^2}) - (\bar{t})^2}$$

$$\text{slope} = \frac{-0.36237 - 0.266667 \times (-)0.53112}{0.091859 - (0.26667)^2} = -10.6387 \; (m/s^2)$$

$$\text{intercept} = B = \frac{(\overline{x^2})(\bar{y}) - (\bar{x})(\overline{xy})}{(\overline{x^2}) - (\bar{x})^2} = \frac{(\overline{t^2})(\bar{V_z}) - (\bar{t})(\overline{V_z t})}{(\overline{t^2}) - (\bar{t})^2}$$

$$\text{intercept} = \frac{0.091859 \times (-)0.53112 - 0.266667 \times (-)0.36237}{0.091859 - (0.266667)^2} = 2.305876 \; (m/s)$$

$$\delta_{slope} = \delta\Delta \sqrt{\frac{1}{N(\overline{x^2} - \bar{x}^2)}} \qquad \delta_{intercept} = \delta\Delta \sqrt{\frac{\overline{x^2}}{N(\overline{x^2} - \bar{x}^2)}}$$

$$\delta\Delta = \sqrt{\frac{\Delta total}{N-2}} \longrightarrow \Delta total = N(\overline{y^2} + A^2\overline{x^2} + B^2 - 2A\,\overline{xy} - 2B\bar{y} + 2AB\bar{x})$$

$$\Delta total = N(\overline{V_z^2} + A^2\overline{t^2} + B^2 - 2A\,\overline{V_z t} - 2B\bar{V_z} + 2AB\bar{t})$$

$$\Delta total = 15(2.644487 + (-10.6387)^2(0.091859) + (2.305876)^2 - 2(-10.6387)(-0.3627)$$
$$-2(2.305876)(-0.53112) + 2(-10.6387)(2.305876)(0.26667)) = 0.210816$$

$$\delta\Delta = \sqrt{\frac{0.210816}{13}} = 0.127345$$

$$\delta_{slope} = (0.127345)\sqrt{\frac{1}{N(\overline{t^2} - \bar{t}^2)}} \qquad \delta_{intercept} = (0.127345)\sqrt{\frac{\overline{t^2}}{N(\overline{t^2} - \bar{t}^2)}}$$

$$\delta_{slope} = (0.127345)\sqrt{\frac{1}{15(0.091859 - (0.266667)^2)}} = 0.228268 \; (m/s^2) \qquad = 0.2$$

$$\delta_{intercept} = (0.127345)\underbrace{\sqrt{\frac{0.091859}{15(0.091859 - (0.266667)^2)}}}_{\text{same as for } V_x \text{ calcs because only } t \text{ here}} = 0.069184 \; (m/s) \qquad = 0.07$$

8. State the initial horizontal and vertical components of velocity and acceleration of the projectile. (Simply state the quantities using appropriate significant figures.)

The initial components of velocity are the components when t = 0, which are just the intercepts of the velocity vs time plots. The uncertainty in these values will just be the uncertainty in the intercepts. Then, we state the values to the same precision as the first digit of the uncertainty:

$V x_i = (1.41 \pm 0.02)$ m/s

$V z_i = (2.31 \pm 0.07)$ m/s

The initial components of acceleration are constant, as estimated by the line of best fit found through least squares analysis. This means that these values are just the slopes of the velocity vs time plots. The uncertainty will just be the uncertainty in the slope. Stating the values to the same precision as the first digit of the uncertainties:

$a_{xi} = (0.73 \pm 0.07)$ m/s

$a_{zi} = (-10.6 \pm 0.2)$ m/s

9. State the expected values of the components of the acceleration. Perform a statistical test for whether your measured acceleration component values agree with your expectations.

The expected value for horizontal acceleration is 0, as we are neglecting air resistance, and therefore there aren't any forces acting sideways on the ball in flight.

The expected value for vertical acceleration, ignoring air resistance, is $-9.8092 m/s^2$, because the only force acting on the ball is gravity, and we are in Victoria.

$$t = \left| \frac{x_1 - x_2}{\sqrt{\delta_{x_1}^2 + \delta_{x_2}^2}} \right| \quad \text{or if } \delta_{x_2} \ll \delta_{x_1} \text{ from } x_2 \text{ being given}: \quad t = \left| \frac{x_1 - x_2}{\delta x_1} \right|$$

For horizontal acceleration:

$$t = \left| \frac{0.73 - 0}{0.07} \right| = 10.5$$

For vertical acceleration:

$$t = \left| \frac{-10.6 - (-9.8092)}{0.2} \right| = 3.63$$

Neither component values agree with the expectations.

10. Respond to the following questions/instructions:

(a) Were any assumptions or approximations involved in performing these calculations? List them and state how you think it might affect the results if each assumption were not valid.

1. We assumed there was no air resistance, which is why the theoretical value for horizontal acceleration is just 0. If air resistance is significant, I would think it would introduce a small horizontal acceleration in the negative x direction, in opposition of the direction of motion. It would also oppose vertical movement, meaning it would make the ball move upwards and downwards slightly slower than theoretically. This may decrease the vertical acceleration value obtained.

The intuitive idea about what would happen to the horizontal acceleration if the air resistance were significant is that it would slow the ball down over time. However, our data points show a consistent and surprisingly large acceleration in the positive x direction, which is in the opposite direction. This is very odd, as I can't think of any unaccounted-for forces on the ball that would accelerate it horizontally. I think the most likely cause of this large error is a mistake / mistakes when manually placing the points onto the video of the projectile, although it is still odd that the x velocities have such a clear upwards trend.

It is also odd that the vertical acceleration calculated is higher than the accepted value, as I theorized that the existence of air resistance would decrease the acceleration of the ball. One thing that could have caused this error is inaccuracy of the scale bar placed in LoggerPro; With the video being very low resolution, the ends of the meter stick reference were very fuzzy. It is feasible that the points selected could have been about 10% closer together than they should've been, which would account for the about 10% difference in scale of the accepted and measured acceleration.

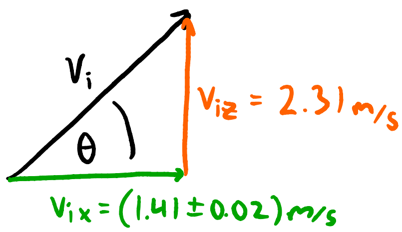
2. The position values were hand selected frame-by-frame on a 320X240 video, which means all the position values are approximate. If these positions were inaccurate, however, they are probably inaccurate in random directions relative to each other, meaning that the overall effect on the trends of the data should be small. This could still be one source of all the error in our results.

3. We assumed that measurement uncertainty would be insignificant relative to statistical uncertainty. This may have had the result of increasing the final t values, because it is possible the measurement uncertainty would have added significantly to the statistical uncertainty.

(b) What do your statistical tests indicate? What are the implications of the results?

Both t values are significantly greater than 2, meaning the results have low accuracy, as the experimental values do not agree with the theoretical values, within the precision of the test. Something either went wrong in the experimental method, the data collection, or the calculations. There is the other option that we've just disproved the laws of physics, but that seems relatively unlikely.

(c) What is the initial speed of your projectile? What is the initial angle from horizontal of the projectile's velocity?



$$|V_i| = \sqrt{2.31^2 + 1.41^2}$$

$$|V_i| = 2.70\ m/s$$

$$\theta = \arctan\left(\frac{2.31}{1.41}\right)$$

$$\theta = 58.6^\circ$$

Initial velocity is 2.70 m/s, 58.6° above horizontal

Note: I didn't propagate uncertainties because we don't have templates for the uncertainties from trig equations.