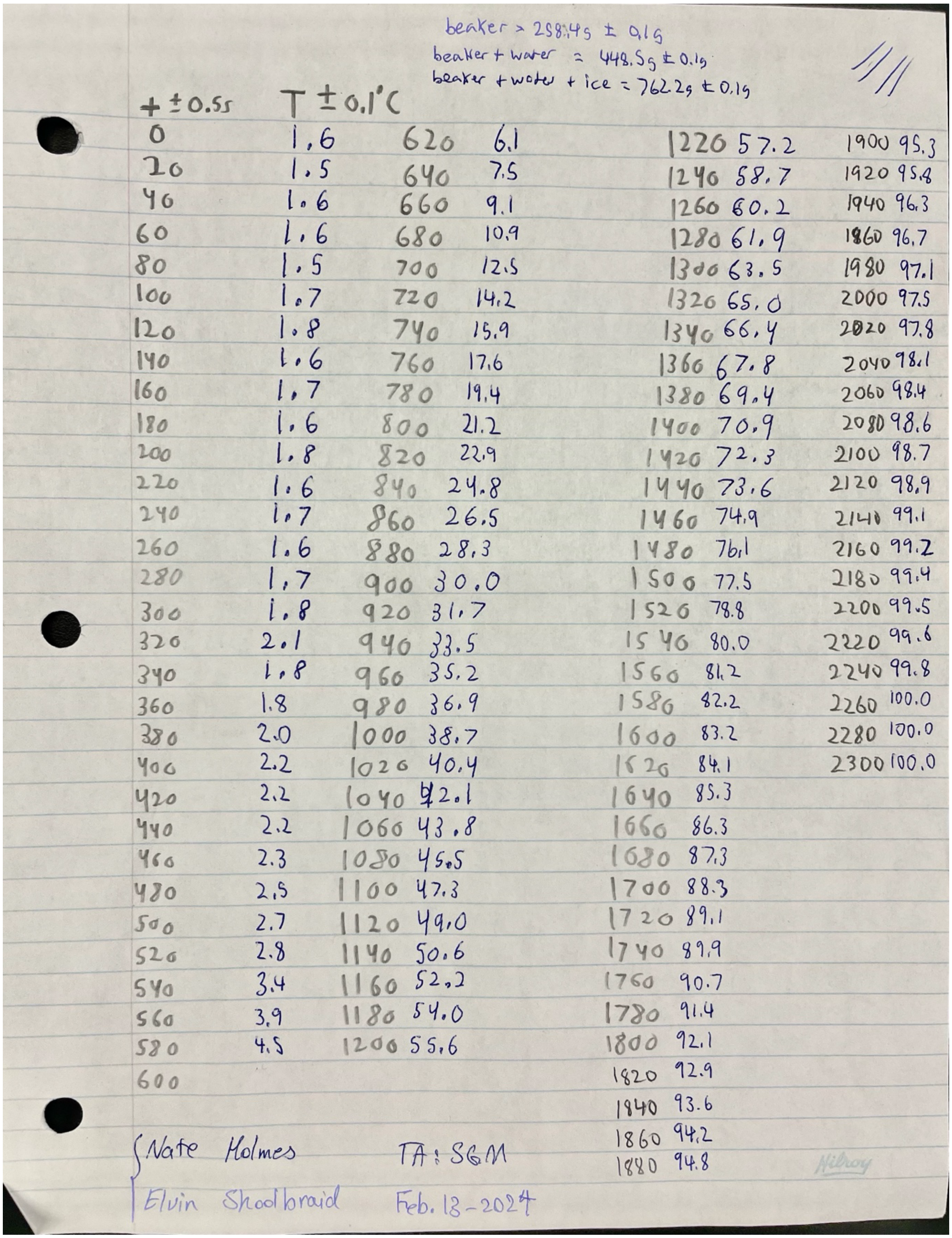
Lab 4 Analysis Worksheet

1. Include a photo of your raw data, including your TA's signature. Failure to include this data will result in a grade of zero for the entire lab report.



beaker = 258.4g ± 0.1g
beaker + water = 448.5g ± 0.1g
beaker + water + ice = 762.2g ± 0.1g

t ± 0.5s	T ± 0.1°C						
0	1.6	620	6.1	1220	57.2	1900	95.3
20	1.5	640	7.5	1240	58.7	1920	95.8
40	1.6	660	9.1	1260	60.2	1940	96.3
60	1.6	680	10.9	1280	61.9	1860	96.7
80	1.5	700	12.5	1300	63.5	1980	97.1
100	1.7	720	14.2	1320	65.0	2000	97.5
120	1.8	740	15.9	1340	66.4	2020	97.8
140	1.6	760	17.6	1360	67.8	2040	98.1
160	1.7	780	19.4	1380	69.4	2060	98.4
180	1.6	800	21.2	1400	70.9	2080	98.6
200	1.8	820	22.9	1420	72.3	2100	98.7
220	1.6	840	24.8	1440	73.6	2120	98.9
240	1.7	860	26.5	1460	74.9	2140	99.1
260	1.6	880	28.3	1480	76.1	2160	99.2
280	1.7	900	30.0	1500	77.5	2180	99.4
300	1.8	920	31.7	1520	78.8	2200	99.5
320	2.1	940	33.5	1540	80.0	2220	99.6
340	1.8	960	35.2	1560	81.2	2240	99.8
360	1.8	980	36.9	1580	82.2	2260	100.0
380	2.0	1000	38.7	1600	83.2	2280	100.0
400	2.2	1020	40.4	1620	84.1	2300	100.0
420	2.2	1040	42.1	1640	85.3		
440	2.2	1060	43.8	1660	86.3		
460	2.3	1080	45.5	1680	87.3		
480	2.5	1100	47.3	1700	88.3		
500	2.7	1120	49.0	1720	89.1		
520	2.8	1140	50.6	1740	89.9		
540	3.4	1160	52.2	1760	90.7		
560	3.9	1180	54.0	1780	91.4		
580	4.5	1200	55.6	1800	92.1		
600				1820	92.9		
				1840	93.6		
				1860	94.2		
				1880	94.8		

Nate Holmes
Elvin Shoolbraid

TA: SGM
Feb. 13-2024

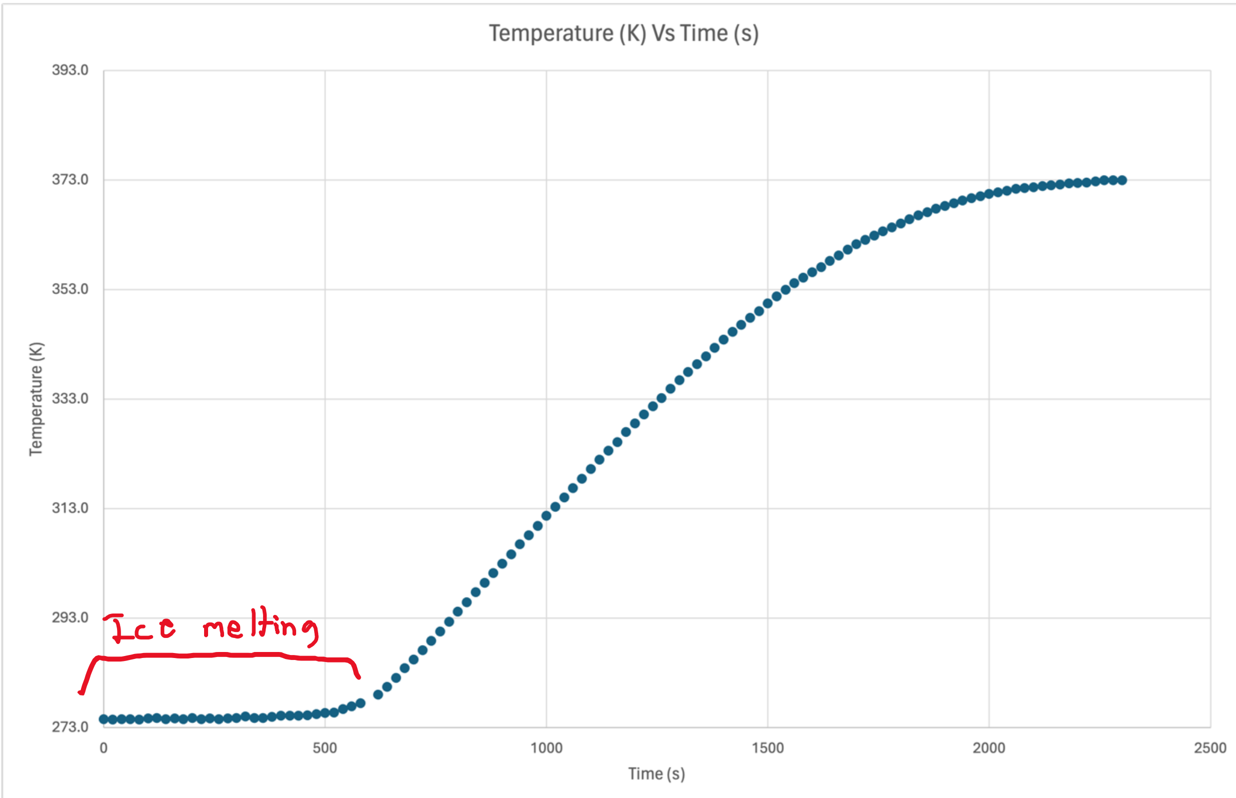
2. Include a table of your time and temperature data.

Time (±0.5s)	**Temperature (±0.1K)**
0	274.6
20	274.5
40	274.6
60	274.6
80	274.5
100	274.7
120	274.8
140	274.6
160	274.7
180	274.6
200	274.8
220	274.6
240	274.7
260	274.6
280	274.7
300	274.8
320	275.1
340	274.8
360	274.8
380	275.0
400	275.2
420	275.2
440	275.2
460	275.3
480	275.5
500	275.7
520	275.8
540	276.4
560	276.9
580	277.5
600	
620	279.1
640	280.5
660	282.1
680	283.9
700	285.5
720	287.2
740	288.9
760	290.6

780	292.4
800	294.2
820	295.9
840	297.8
860	299.5
880	301.3
900	303.0
920	304.7
940	306.5
960	308.2
980	309.9
1000	311.7
1020	313.4
1040	315.1
1060	316.8
1080	318.5
1100	320.3
1120	322.0
1140	323.6
1160	325.2
1180	327.0
1200	328.6
1220	330.2
1240	331.7
1260	333.2
1280	334.9
1300	336.5
1320	338.0
1340	339.4
1360	340.8
1380	342.4
1400	343.9
1420	345.3
1440	346.6
1460	347.9
1480	349.1
1500	350.5
1520	351.8
1540	353.0
1560	354.2
1580	355.2
1600	356.2

1620	357.1
1640	358.3
1660	359.3
1680	360.3
1700	361.3
1720	362.1
1740	362.9
1760	363.7
1780	364.4
1800	365.1
1820	365.9
1840	366.6
1860	367.2
1880	367.8
1900	368.3
1920	368.8
1940	369.3
1960	369.7
1980	370.1
2000	370.5
2020	370.8
2040	371.1
2060	371.4
2080	371.6
2100	371.7
2120	371.9
2140	372.1
2160	372.2
2180	372.4
2200	372.5
2220	372.6
2240	372.8
2260	373.0
2280	373.0
2300	373.0

3. Include a plot of your temperature versus time data and identify on the plot the region where the ice is melting.



4. Include a small table of the four cells of the LINEST calculation of the slope and intercept of the temperature-time data in the appropriate regime. Use these results to calculate the rate of heat gain by the water, along with its uncertainty.

Slope (K/s):	0.087867133	**Intercept (K):**	223.9109557
Uncertainty:	0.000252459	**Uncertainty:**	0.205233682

$$\text{Mass}_{(\text{beaker + water + ice})} = (762.2 \pm 0.1)\,g$$

$$\text{Mass}_{\text{beaker}} = (258.4 \pm 0.1)\,g$$

$$\text{Mass}_{(\text{melted water})} = \text{Mass}_{(\text{beaker + water + ice})} - \text{Mass}_{\text{beaker}}$$

$$= 0.5038\,kg \longrightarrow \;= (0.5038 \pm 0.0001)\,kg$$

$$\delta_{\text{mass}_{(\text{melted water})}} = \sqrt{(0.1g)^2 + (0.1g)^2} = \frac{\sqrt{2}}{10}g = \frac{\sqrt{2}}{10000}\,kg$$

Specific heat capacity of water $= 4184\ J \cdot kg^{-1} \cdot K^{-1}$

We have:

$$4184\left[\frac{J}{kg \cdot K}\right],\quad 0.5038[kg],\quad 0.0879\left[\frac{K}{s}\right]$$

We want:

$$\frac{dQ}{dt}\left[J/s\right]$$

We find:

$$\left[\frac{J}{kg \cdot K}\right]\cdot[kg]\cdot\left[\frac{K}{s}\right] = [J/s] = [W]$$

$$\therefore\ \frac{dQ}{dt} = \left(4184\,\frac{J}{kg \cdot K}\right)(0.5038\,kg)\left(0.087867133\,\frac{K}{s}\right)$$

$$= 185.215059\ J/s$$

$$\delta_{\frac{dQ}{dt}} = \left|\frac{dQ}{dt}\right|\sqrt{\left(\frac{\delta_m}{m}\right)^2 + \left(\frac{\delta_{\text{slope}}}{\text{slope}}\right)^2 + \cancelto{0}{\left(\frac{\delta_{\text{Heat capacity}}}{\text{Heat capacity}}\right)^2}}$$

$$= 185.2\sqrt{\left(\frac{0.0001}{0.5038}\right)^2 + \left(\frac{0.00025}{0.0879}\right)^2} = 0.535\ J/s$$

$$\text{Heat rate} = (185.2 \pm 0.5)\ J/s$$

5. Show the derivation of how you can calculate the latent heat of fusion of water from the rate of heat gain, and use the formula to calculate the latent heat of fusion of water, along with its uncertainty.

time to melt ice $\approx$ 580s

$$\left(\frac{dQ}{dt}\right)(\Delta t) = \Delta Q\,[J]$$

$$M_{ice} = M_{beaker+water+ice} - M_{beaker+water} = (0.3137 \pm 0.0001)\,kg$$

Latent heat of fusion of water : $\left[\frac{J}{kg}\right]$ so $= \frac{\Delta Q}{m}$

$$\frac{\frac{dQ}{dt} \cdot \Delta t}{m} = \frac{(185.2150599\,J/s)(580s)}{0.3137\,kg} = 342{,}439.4\ J/kg$$

$$\delta = \left|\text{Latent heat}\right| \sqrt{\left(\frac{0.5}{185.2}\right)^2 + \left(\frac{0.5s}{580s}\right)^2 + \left(\frac{0.0001}{0.3137}\right)^2} = 976.6\ J/kg$$

Latent heat $= (342{,}000 \pm 1000)\ J/kg = (3.42 \pm 0.01) \times 10^5\ J/kg$

6. Perform a statistical comparison between your determined value of L versus the accepted value of L.

$$t = \left| \frac{L_{calc} - L_{accepted}}{\sqrt{\delta_{L_{calc}}^2 + \cancel{\delta_{L_{accepted}}^2}}} \right| = \left| \frac{342{,}439.4 - 3.33\times10^5}{976.6} \right| = 9.67$$

7. Include a table of the temperature, log of temperature, central-difference slope of temperature, instantaneous net heat gain, the rate of heat loss, and the log of the rate of heat loss in the region where T > 50°C. You do not need uncertainties for these columns.

$H_{net} = mc(dT/dt) = (0.5038kg)(4184J/K)(dT/dt)$

$H_{loss} = H_{gain} - H_{net} = 185.22J/s - H_{net}$

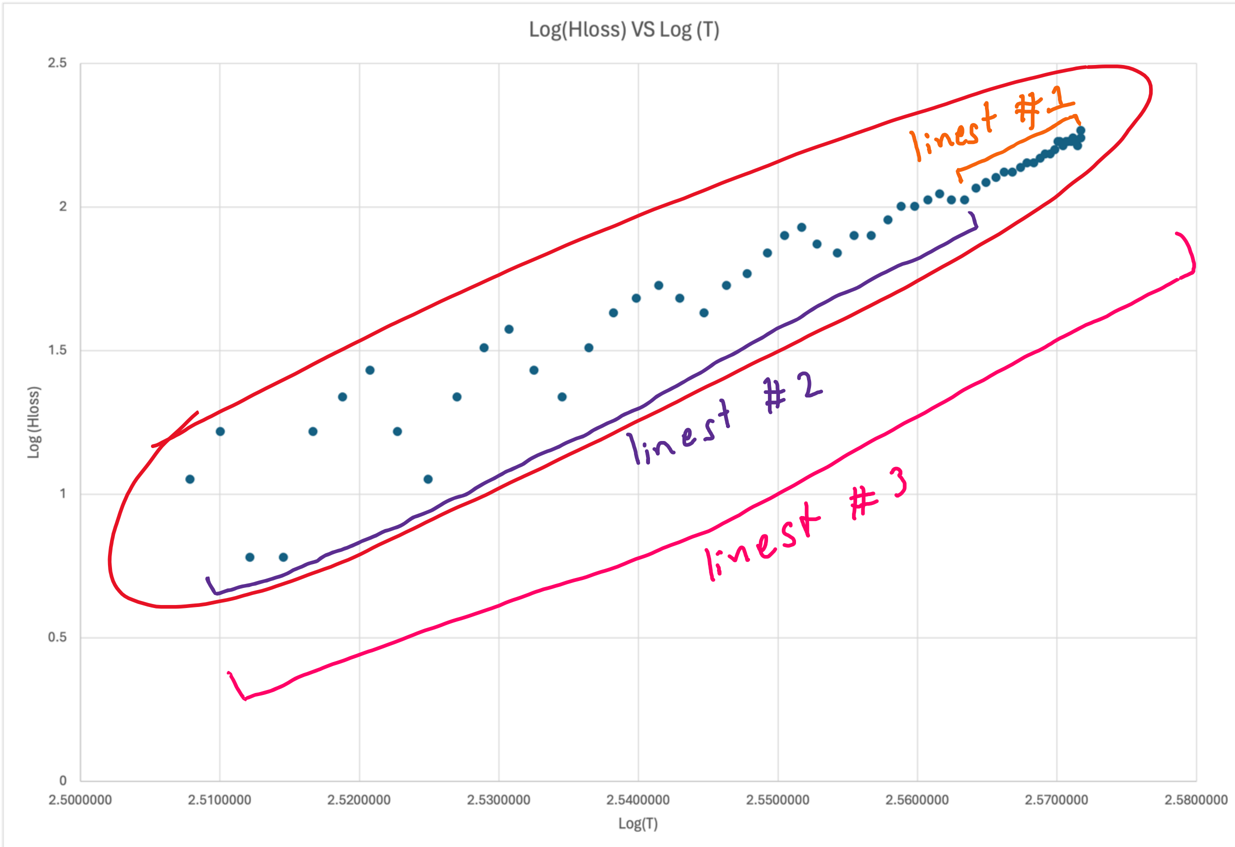
Starting at T > 50C → T > 323K:

T (K)	log(T)	dT/dt (K/s)	H_{net} (J)	H_{loss} (J)	log(H_{loss})
323.6	2.5100085	0.08	168.631936	16.58312308	1.219666324
325.2	2.5121505	0.085	179.171432	6.043627077	0.781297658
327.0	2.5145478	0.085	179.171432	6.043627077	0.781297658
328.6	2.5166676	0.08	168.631936	16.58312308	1.219666324
330.2	2.5187771	0.0775	163.362188	21.85287108	1.339508504
331.7	2.5207455	0.075	158.09244	27.12261908	1.433331625
333.2	2.5227050	0.08	168.631936	16.58312308	1.219666324
334.9	2.5249151	0.0825	173.901684	11.31337508	1.053592186
336.5	2.5269851	0.0775	163.362188	21.85287108	1.339508504
338.0	2.5289167	0.0725	152.822692	32.39236708	1.510442685
339.4	2.5307118	0.07	147.552944	37.66211508	1.575904706
340.8	2.5324996	0.075	158.09244	27.12261908	1.433331625
342.4	2.5345338	0.0775	163.362188	21.85287108	1.339508504
343.9	2.5364322	0.0725	152.822692	32.39236708	1.510442685
345.3	2.5381966	0.0675	142.283196	42.93186308	1.632779736
346.6	2.5398286	0.065	137.013448	48.20161108	1.683061554
347.9	2.5414544	0.0625	131.7437	53.47135908	1.728121223
349.1	2.5429498	0.065	137.013448	48.20161108	1.683061554
350.5	2.5446880	0.0675	142.283196	42.93186308	1.632779736
351.8	2.5462958	0.0625	131.7437	53.47135908	1.728121223
353.0	2.5477747	0.06	126.473952	58.74110708	1.768942127
354.2	2.5492486	0.055	115.934456	69.28060308	1.840611659
355.2	2.5504730	0.05	105.39496	79.82009908	1.902112263
356.2	2.5516939	0.0475	100.125212	85.08984708	1.929877743
357.1	2.5527899	0.0525	110.664708	74.55035108	1.872449693
358.3	2.5542468	0.055	115.934456	69.28060308	1.840611659
359.3	2.5554572	0.05	105.39496	79.82009908	1.902112263
360.3	2.5566643	0.05	105.39496	79.82009908	1.902112263
361.3	2.5578680	0.045	94.855464	90.35959508	1.955974276
362.1	2.5588285	0.04	84.315968	100.8990911	2.003887254

362.9	2.5597870	0.04	84.315968	100.8990911	2.003887254
363.7	2.5607433	0.0375	79.04622	106.1688391	2.025997069
364.4	2.5615784	0.035	73.776472	111.4385871	2.047035597
365.1	2.5624118	0.0375	79.04622	106.1688391	2.025997069
365.9	2.5633624	0.0375	79.04622	106.1688391	2.025997069
366.6	2.5641925	0.0325	68.506724	116.7083351	2.067101874
367.2	2.5649027	0.03	63.236976	121.9780831	2.086281804
367.8	2.5656117	0.0275	57.967228	127.2478311	2.104650389
368.3	2.5662017	0.025	52.69748	132.5175791	2.122273493
368.8	2.5667909	0.025	52.69748	132.5175791	2.122273493
369.3	2.5673793	0.0225	47.427732	137.7873271	2.139209275
369.7	2.5678495	0.02	42.157984	143.0570751	2.155509341
370.1	2.5683191	0.02	42.157984	143.0570751	2.155509341
370.5	2.5687882	0.0175	36.888236	148.3268231	2.171219695
370.8	2.5691397	0.015	31.618488	153.5965711	2.186381521
371.1	2.5694910	0.015	31.618488	153.5965711	2.186381521
371.4	2.5698419	0.0125	26.34874	158.8663191	2.201031833
371.6	2.5700757	0.0075	15.809244	169.4058151	2.228928314
371.7	2.5701926	0.0075	15.809244	169.4058151	2.228928314
371.9	2.5704262	0.01	21.078992	164.1360671	2.215204023
372.1	2.5706597	0.0075	15.809244	169.4058151	2.228928314
372.2	2.5707764	0.0075	15.809244	169.4058151	2.228928314
372.4	2.5710097	0.0075	15.809244	169.4058151	2.228928314
372.5	2.5711263	0.005	10.539496	174.6755631	2.242232152
372.6	2.5712429	0.0075	15.809244	169.4058151	2.228928314
372.8	2.5714759	0.01	21.078992	164.1360671	2.215204023
373.0	2.5717088	0.005	10.539496	174.6755631	2.242232152
373.0	2.5717088	0	0	185.2150591	2.267676294
373.0	2.5717088				

8. Include a plot of your $\log(H_{loss})$ vs $\log(T)$ data and identify regions that are approximately linear.

   All the data from the range of the table appears rather linear apart from a decrease in distribution range over time:



9. Use the LINEST function on the data in the regions that are approximately linear, and use this data to determine the power of the temperature dependence in each region along with the coefficient of heat loss in the model $H_{loss} = AT^n$. Include a table of the LINEST data for each region you calculate a slope, and clearly identify the data you are using in your analysis.

   Linest #1: For log(T) > 2.54:

24.14372408	-59.84227917
1.006361984	2.585428525

   Linest #2: For log(T) < 2.54:

20.07504926	-49.37651586
1.232920085	3.130676636

   Linest #3: For All T > 50C data:

19.82451193	-48.74336839
0.619473306	1.580476445

$$H_{loss} = AT^{n}$$

$$\log(H_{loss}) = \log(AT^{n}) = n\log(T) + \log(A)$$

$$y = mx + b$$

n is the slope of the graph

& $\log(A)$ is the intercept

so $A = 10^{\log(A)} = 10^{\log(\text{intercept})}$

So, $n = (24 \pm 1)$, (20 ± 1), and (19.8 ± 0.6) for LINEST #1, #2, & #3, respectively,

& $A = 1.44 * 10^{-60}$, $4.20 * 10^{-50}$, & $1.81 * 10^{-49}$.

10. Respond to the following questions/instructions:

(a) For many groups, the temperature where the ice is melting will not be zero. Regardless of what your temperatures were as the ice was melting, what are some possible reasons why this might be the case, and why does it not have an impact on the analysis in this lab activity?

The temperature displayed may not have been 0 because of innacurate calibration of the thermometers, seeing as they were simple household thermometers that didn't undergo any manual calibration in the lab.

This doesn't impact the analysis partly because many calculations (numerical derivatives, slopes, heat rate, time of melting) only rely on *relative* temperature values, meaning the miscalibration has no effect. For calculations where the absolute temperature is used, a difference less than 2 degrees will cause less than a 1% error margin, which is likely less than other sources of error.

(b) What are some aspects of the experimental design that may explain why your value of L may differ from the accepted value?

The mass used for the latent heat calculation is the mass of ice that was added to the beaker. However, this ice was not at exactly 0C, and was likely a few degrees below, because having ice at exactly zero would presumably be difficult. Therefore, if negligible ice melted after being added to the water, the latent heat estimate would bias higher because the calculation is actually measuring the energy required to bring the average temperature in the beaker up to 0C and then melt the ice. If significant ice melted after being added to the water, the actual mass of ice being melted from 0C would be less, and the estimate would bias lower. The amount of ice that melted to bring the water to 0C depends on the relative weights and temperatures of the water and ice.

A more accurate estimate could be found by measuring the weight of ice once it had been brought up to just below zero by the water, however it would be difficult to insulate the ice through that process. Another option would be to measure the temperature of the ice and water before mixing, and calculate the required mass ratio to bring the mixture to exactly 0C before the experiment.

(c) What forms of heat loss did you find dominated the heat loss at high temperatures? Justify your answer. If you cannot clearly identify any particular form of heat loss, explain why this might be.

The calculated n values, between 20 and 24, are much higher than any of the ranges talked about in the lab manual and therefore the only candidate is evaporative heat loss, because an n value greater than 4 is possible then.

(d) Explain why using a least squares approach applied to H_{loss} vs T data would not provide useful results.

Least squares analysis tries to find the most accurate straight line to follow the data, meaning it is only useful for linear data. H_{loss} vs T is an exponential relationship, meaning a curved graph that needs to be “linearized” before it can be analysed like a linear function. Taking the log of both H and T makes the graph trend a linear one, so without doing that, the LINEST function would not produce useful or insightful results.