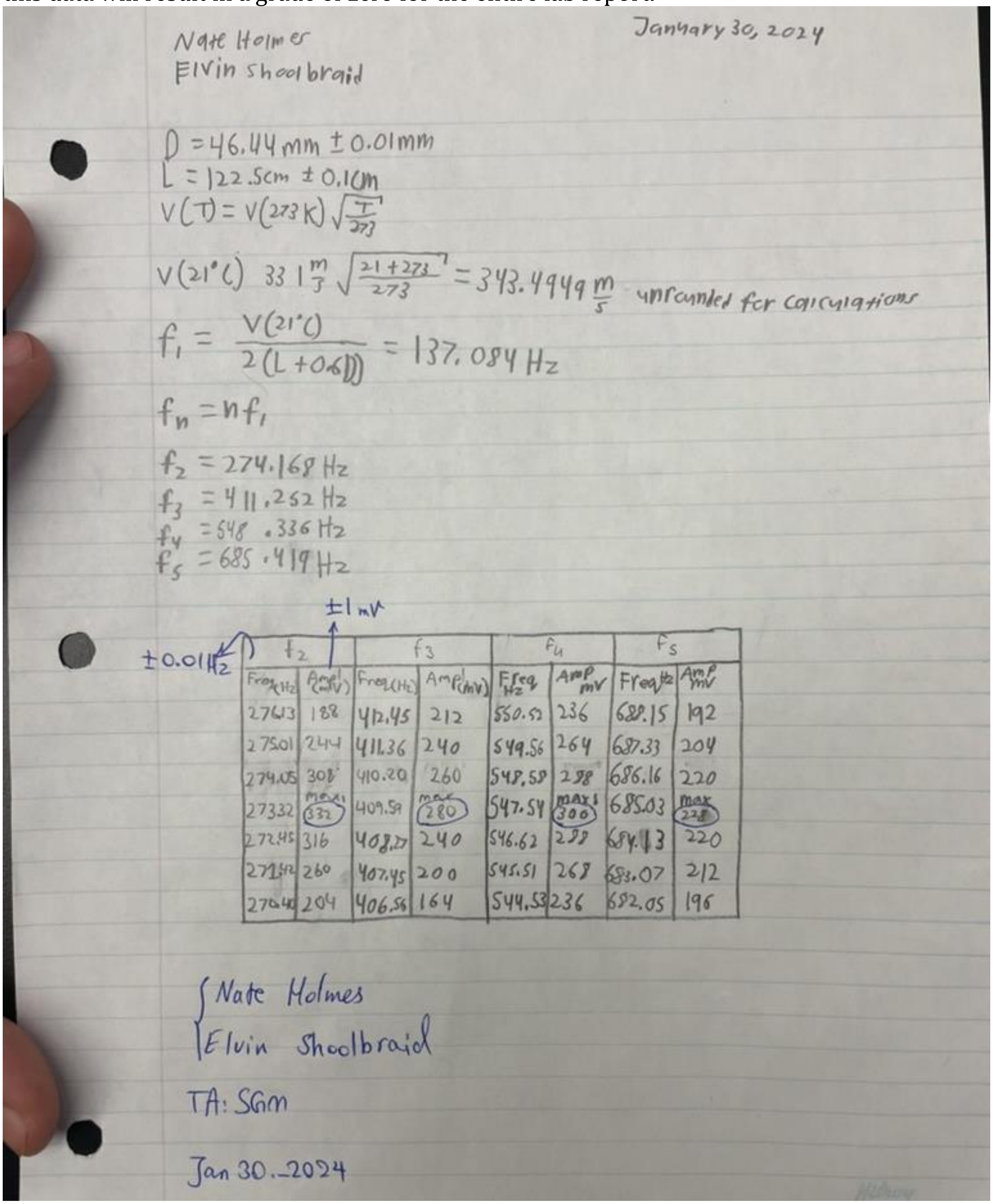
Lab 3 Analysis Worksheet

1. Include a photo of your raw work, including your TA's signature. Failure to include this data will result in a grade of zero for the entire lab report.



January 30, 2024

Nate Holmes
Elvin Shoolbraid

$D = 46.44\,mm \pm 0.01\,mm$

$L = 122.5\,cm \pm 0.1\,cm$

$v(T) = v(273\,K)\sqrt{\frac{T}{273}}$

$v(21°C)\ 331\frac{m}{s}\sqrt{\frac{21+273}{273}} = 343.4949\,\frac{m}{s}$ unrounded for calculations

$f_1 = \frac{v(21°C)}{2(L+0.6D)} = 137.084\,Hz$

$f_n = nf_1$

$f_2 = 274.168\,Hz$

$f_3 = 411.252\,Hz$

$f_4 = 548.336\,Hz$

$f_5 = 685.419\,Hz$

±0.01 Hz ±1 mV

f_2		f_3		f_4		f_5	
Freq (Hz)	Amp (mV)	Freq (Hz)	Amp (mV)	Freq Hz	Amp mV	Freq Hz	Amp mV
276.13	188	412.45	212	550.52	236	688.15	192
275.01	244	411.36	240	549.56	264	687.33	204
274.05	308	410.20	260	548.58	298	686.16	220
273.32	max! 332	409.59	max 280	547.54	max! 300	685.03	max 228
272.45	316	408.27	240	546.62	288	684.13	220
271.42	260	407.45	200	545.51	268	683.07	212
270.40	204	406.56	164	544.53	236	682.05	196

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TA: SGm

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2. Calculate the theoretical value of the fundamental frequency and its uncertainty.

$$f_n = n\left(\frac{V}{2(L+2d)}\right)$$

$$V(T) = \underbrace{V(273K)}_{=331\,m/s}\sqrt{\frac{T}{273}}$$

T: temp (°K)

$n=1$, $L=(0.1225 \pm 0.001)m$,

$d = (0.04644 \pm 0.0001)m$

$21°c = 294°K$

$$V(294°K) = (331\,m/s)\sqrt{\frac{294}{273}} = V = 343.4949\,m/s$$

$$f_1 = 137.084\,Hz$$

Uncertainty:

$$f_1 = \frac{V}{2(L+2d)} = \frac{V(273K)\sqrt{\frac{T}{273}}}{2(L+2d)}$$

$$\delta_{f_1} = \sqrt{\left(\frac{\partial f_1}{\partial T}\delta_T\right)^2 + \left(\frac{\partial f_1}{\partial L}\delta L\right)^2 + \left(\frac{\partial f_1}{\partial d}\delta_d\right)^2}$$

$$\frac{\partial}{\partial T}\frac{V(273k)\sqrt{\frac{T}{273}}}{2(L+2d)} = \frac{V(273K)}{2(L+2d)\sqrt{273}}\frac{\partial}{\partial T}\sqrt{T} = \frac{V(273K)}{2(L+2d)\sqrt{273}}\cdot\frac{1}{2\sqrt{T}} = \frac{f_1}{2T}$$

$$\frac{\partial}{\partial L}f_1 = -\frac{V(273k)\sqrt{\frac{T}{273}}}{2(L+2d)^2} = \frac{-f_1}{L+2d}$$

$$\frac{\partial}{\partial d}f_1 = -\frac{V(273k)\sqrt{\frac{T}{273}}}{(L+2d)^2} = \frac{-2f_1}{L+2d}$$

$$\delta_{f_1} = \sqrt{\left(\frac{f_1}{2T}\delta_T\right)^2 + \left(\frac{-f_1}{L+2d}\delta L\right)^2 + \left(\frac{-2f_1}{L+2d}\delta_d\right)^2} = 0.6897\,Hz$$

$L = 0.1225m$, $\delta_L = 0.001m$

$d = 0.04644m$, $\delta_d = 0.0001m$

$T = 294°K$, $\delta_T = 1°K$

$V(273K) = 331\,m/s$, $\delta_{V(273K)} = 0$

So

$$f_1 = (137.1 \pm 0.7)\,Hz$$

3. Include your four tables of frequency f_i, peak-to-peak amplitude F_i, and central-difference derivative of the amplitude F_i'. Remember, the central difference method cannot be used to calculate the derivative for the first and last entries.

f_2 theoretical = 274.168 Hz

Frequency (± 0.01Hz)	Amplitude (± 1mV)	Deriv. Ampl. (mV/Hz)
276.13	188	
275.01	244	-57.69230769
274.05	308	-52.07100592
273.32	332	-5
272.45	316	37.89473684
271.42	260	54.63414634
270.4	204	

f_3 theoretical = 411.252 Hz

Frequency (± 0.01Hz)	Amplitude (± 1mV)	Deriv. Ampl. (mV/Hz)
412.45	212	
411.36	240	-21.33333333
410.2	260	-22.59887006
409.59	280	10.3626943
408.27	240	37.38317757
407.45	200	44.44444444
406.56	164	

f_4 theoretical = 548.336 Hz

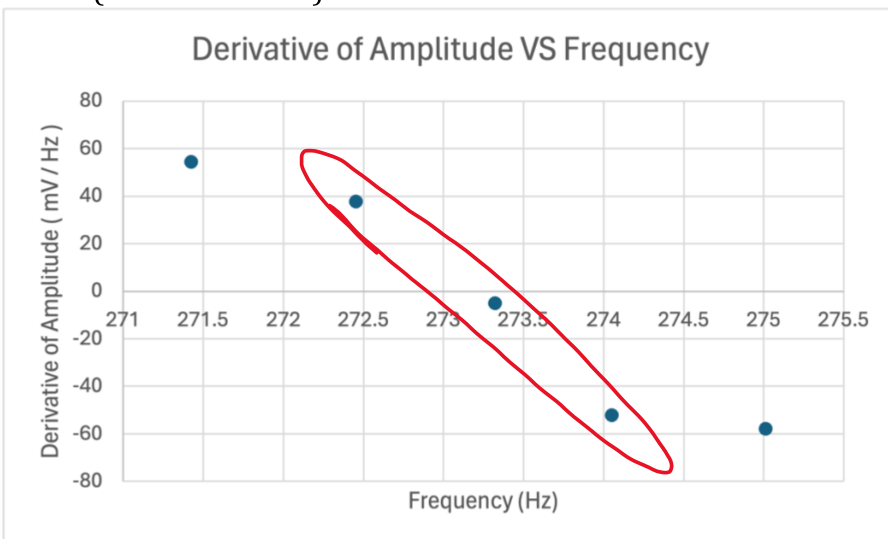
Frequency (± 0.01Hz)	Amplitude (± 1mV)	Deriv. Ampl. (mV/Hz)
550.52	236	
549.56	264	-26.80412371
548.58	288	-17.82178218
547.54	300	0
546.62	288	15.7635468
545.51	268	24.88038278
544.53	236	

f_5 theoretical = 685.419 Hz

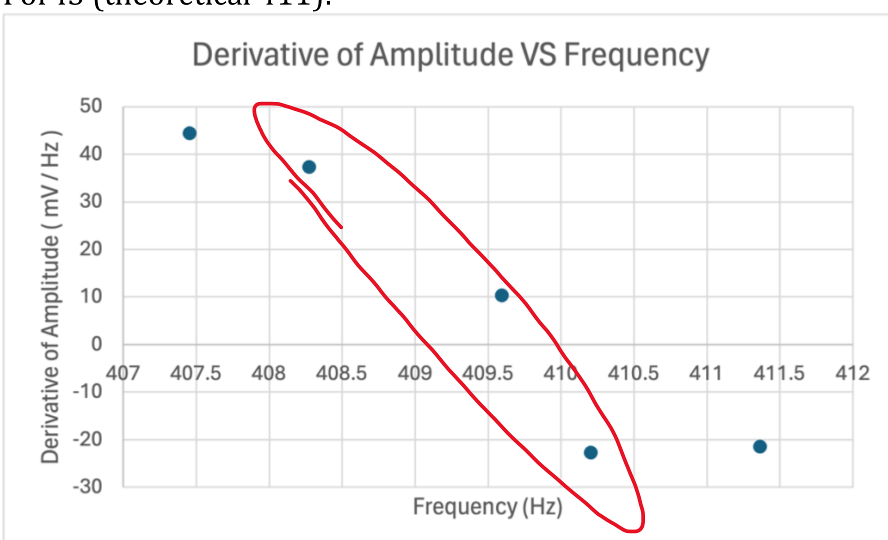
Frequency (± 0.01Hz)	Amplitude (± 1mV)	Deriv. Ampl. (mV/Hz)
688.15	192	
687.33	204	-14.07035176
686.16	220	-10.43478261
685.03	228	0
684.13	220	8.163265306
683.07	212	11.53846154
682.05	196	

4. Produce plots of the finite-difference derivative value versus the frequency value for each of the tables. Clearly identify on the plots which points you feel lie within the central straight-line region. It will be these points that you use with the LINEST function in the next question.

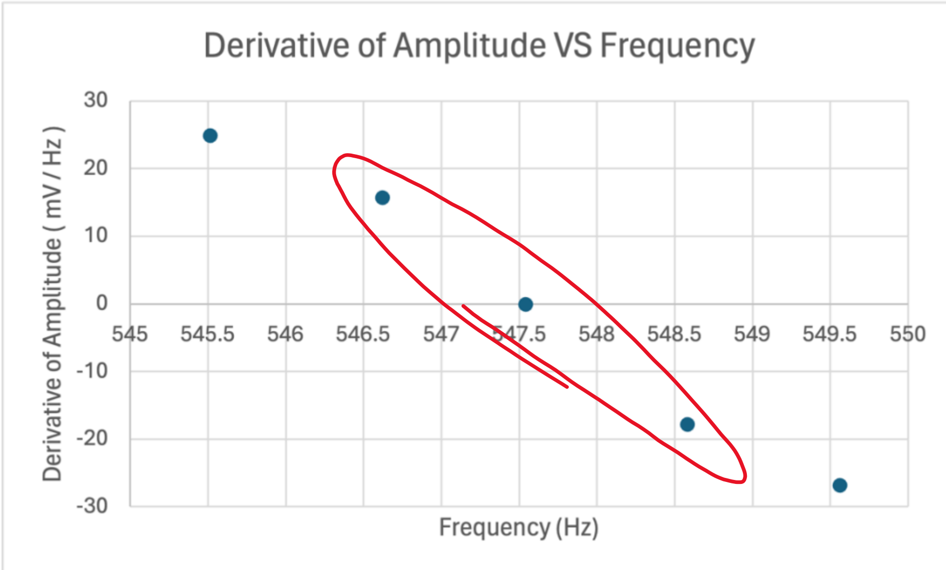
For f2 (theoretical 274):



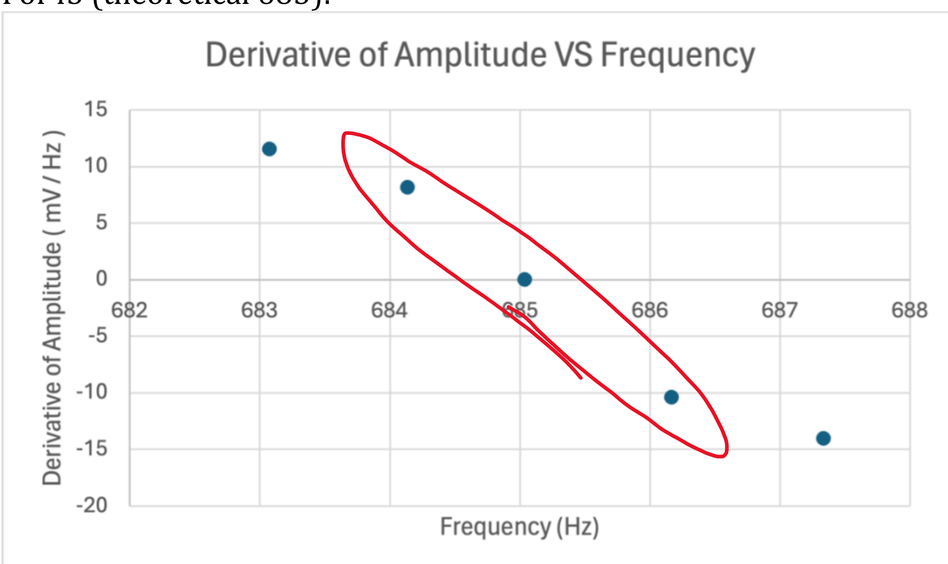
For f3 (theoretical 411):



For f4 (theoretical 548):



For f5 (theoretical 685):



5. Use the LINEST function to calculate the slope and intercept for the F_i' vs f_i data for ONE of the tables. Include a screenshot of your LINEST function output, and make a statement of the slope and intercept, along with its uncertainty, summarizing the results. Since y = Ax + B, the x-intercept occurs at x = −B/A. Use this to calculate the x-intercept and its uncertainty for this table.

For f_2 (theoretical = 274 Hz):

Linest:	-56.009518	15299.5157
	4.33647847	1185.04732

Slope = (-56 ± 4) mV/Hz^2 , Intercept = (15000 ± 1000) mV/Hz

x-intercept = -15299.5157 / -56.009518 = 273.159208 Hz

$\delta_{x\text{-incercept}}$ = (x-intercept) * SQRT[(δ_{slope} / slope)^2 + ($\delta_{intercept}$ / intercept)^2]

= (273.15920) * SQRT (4.33647846/-56.0095183)^2 + (1185.04731/15299.5156)^2
= 29.915588 Hz

Final Statement: x-intercept = (270 ± 30) Hz

6. Now invert the LINEST analysis - calculate the slope of f_i vs F_i' for all four tables. Include a screenshot of your LINEST function output for each one, and accompany each screenshot with a statement of the slope and intercept, along with its uncertainty, summarizing the results.

For f_2 (theoretical 274):

ReverseLinest	
-0.0177477	273.159888
0.0013741	0.05124486

Slope = (-0.018 ± 0.001) mV^{-1} , Intercept = (273.16 ± 0.05) Hz

For f_3 (theoretical 411):

ReverseLinest	
-0.0316819	409.618901
0.00864576	0.22410121

Slope = (-0.032 ± 0.009) mV-1 , Intercept = (409.6 ± 0.2) Hz

For f_4 (theoretical 548):

ReverseLinest	
-0.0583587	547.539961
1.9947E-06	2.74E-05

Slope = (-0.05835 ± 0.00002) mV-1 , Intercept = (547.5400 ± 0.0003) Hz

For f_5 (theoretical 685):

ReverseLinest	
-0.1091122	685.02405
0.00055413	0.00423859

Slope = (-0.1091 ± 0.0006) mV-1 , Intercept = (685.024 ± 0.004) Hz

7. Calculate f_1 for each of the overtone frequencies you determined above.

$f_n = n*f_1 \rightarrow f_1 = f_n / n$

$\delta_{f1} = \delta_{fn} / n$

$f_1 = f_2 / 2$ = (273.159888 / 2 ± 0.05124486 / 2) Hz = (136.57 ± 0.03) Hz

$f_1 = f_3 / 3$ = (136.5 ± 0.1) Hz

$f_1 = f_4 / 4$ = (136.884990 ± 0.000007) Hz

$f_1 = f_5 / 5$ = (137.0048 ± 0.0008) Hz

8. Use these to calculate $f_1 \pm \delta_{f_1}$.

* unrounded values used for calculations

$$f_1 = \frac{136.57^* Hz + 136.5 Hz^* + 136.9 Hz^* + 137.0 Hz^*}{4}$$

$$= 136.7399475 Hz$$

"f_{12}" = f_1 calculated from f_2

$$\delta_{f_1} = \sigma_{\bar{f}_1} \approx \sigma_{f_1}/\sqrt{N} = \frac{\sqrt{\frac{(f_{12}-\bar{f}_1)^2+\cdots+(f_{15}-\bar{f}_1)^2}{(N-1)}}}{\sqrt{N}}$$

$$= \frac{\sqrt{\frac{(136.6^*-136.7^*)^2+\cdots+(137.0^*-136.7^*)^2}{(3)}}}{\sqrt{4}} = 0.12166934 Hz$$

so $f_1 = (136.7 \pm 0.1)$ Hz

9. Perform a statistical comparison of f_1 as compared to your theoretical value.

$$t = \left| \frac{(f_{1\text{ from harmonics}}) - (f_{1\text{ theoretical}})}{\sqrt{(\delta_{f_1 \text{harmon.}})^2 + (\delta_{f_1 \text{theo.}})^2}} \right| = \left| \frac{(136.7399475\,Hz) - (137.084\,Hz)}{\sqrt{(0.12166934\,Hz)^2 + (0.6897\,Hz)^2}} \right|$$

$$t = 2.46$$

Because t is slightly greater than 2, the two results are found to be statistically inconsistent, and cannot be measuring the same quantity, to the precision of the experiment.

10. Respond to the following questions/instructions using sentences:
    (a) Based on your results, does the theoretical value hold up to experimental evidence? Explain how you know.

    The theoretical value *almost* agrees with the experimental evidence, however technically it does not because the t value measuring the similarity of the two values in relation to the total uncertainty of the experiment is greater than 2. Practically, it is a relatively good predictor, but there are clearly environmental factors being ignored to calculate the theoretical value.

    (b) What is the largest source of uncertainty in the experimental determination of f_1? Justify your answer. How might you improve this uncertainty if you were to redesign the experiment?

    The largest source of uncertainty in the experimental determination is likely not the statistical uncertainty, because the statistical uncertainty is small. The uncertainty in the frequency emmited by the wave generator is likely the largest source of uncertainty in the procedure, because of the high sensitivity of the frequency control knob; Left untouched, the frequency would randomly change all the time, and would fall over time. Even with maximal care taken to stabilize the knob, the random oscillations in frequency were likely much greater than the 0.01Hz measurement uncertainty, resulting in unnacounted-for error in this report.

    The primary change I would make to improve the uncertainty in the experiment would be to use a more stable wave generator, that could be precisely controlled and would stay at one frequency on its own.

    (c) Why were you asked to invert the x and y values for using LINEST to calculate the frequencies? What about the analysis in Q4 above suggests that there is a better way to perform the analysis?

LINEST outputs the slope, and the y-intercept of a linear graph, which is the value of the vertical axis at the point where the x axis value is zero. For our graphs with frequency on the x axis and the rate of change of amplitude with respect to frequency on the y-axis, the y-intercept represents the theoretical rate of change of amplitude when the frequency is zero, which isn't a useful quantity. However, swapping the x and y axes means that the "y-intercept" is actually the x intercept of the original graph, which is visible as the point of interest on the graphs in Q4. On these graphs, the x-intercept represents the frequency at which the rate of change of amplitude is zero, which means that the amplitude is at a maximum, and that frequency can be taken as the resonant frequency.

(d) Given the information in the Background section for this lab, what are some possible reasons why the theoretical value and the experimental value might disagree? Explain using math.

One reason they might disagree is that we only estimated the temperature of the labs to be 21 degrees C, and variations could cause 0.3% differences in the results as explained in the lab manual.
The background section also mentions the low number of data points in the lab measurements as a compromise from a more ideal procedure of measuring more points to get a smoother graph. The low number of data points decreases the certainty in the answer and increases the likelyhood of disagreement between theoretical and tested values.

(e) The boundary layer at either end of the tube is theorized to be 0.3D. Given your experimental value of f_1, how big is the boundary layer, relative to D, according to your data?

$$f_1 = \frac{v(21°C)}{2(L + xD)}$$

x = size of boundary layer relative to D

$$L + xD = \frac{v(21°C)}{2f_1} \qquad x = \frac{\frac{v(21°C)}{2f_1} - L}{D}$$

$x = 0.668$ so boundary layer is 0.334m